Teaching Waldorf Mathematics in Grades 1–8

Engaging the maths genius in every child

Ron Jarman

Hawthorn Press

Published by Hawthorn Press,
Hawthorn House, 1 Lansdown Lane, Stroud, Gloucestershire, GL5 1BJ
Tel: (01453) 757040   Email: info@hawthornpress.com
www.hawthornpress.com

Edited by Matthew Barton
Second Edition updated by Stephen Sagarin
Cover image by Abigail Large
Typeset in Plantin by Winslade Graphics
Printed by Severnprint Ltd, Gloucestershire

Acknowledgements
The author would like to thank Barbara Low and Jonathan Swann for reading through the script and making suggestions and the latter for checking answers to examples – but for any remaining errors the author is to blame. The author would also particularly like to thank Elisa Wannert for her cartoons.

British Library Cataloguing in Publication data applied for.

ISBN: 978-1-912480-25-8

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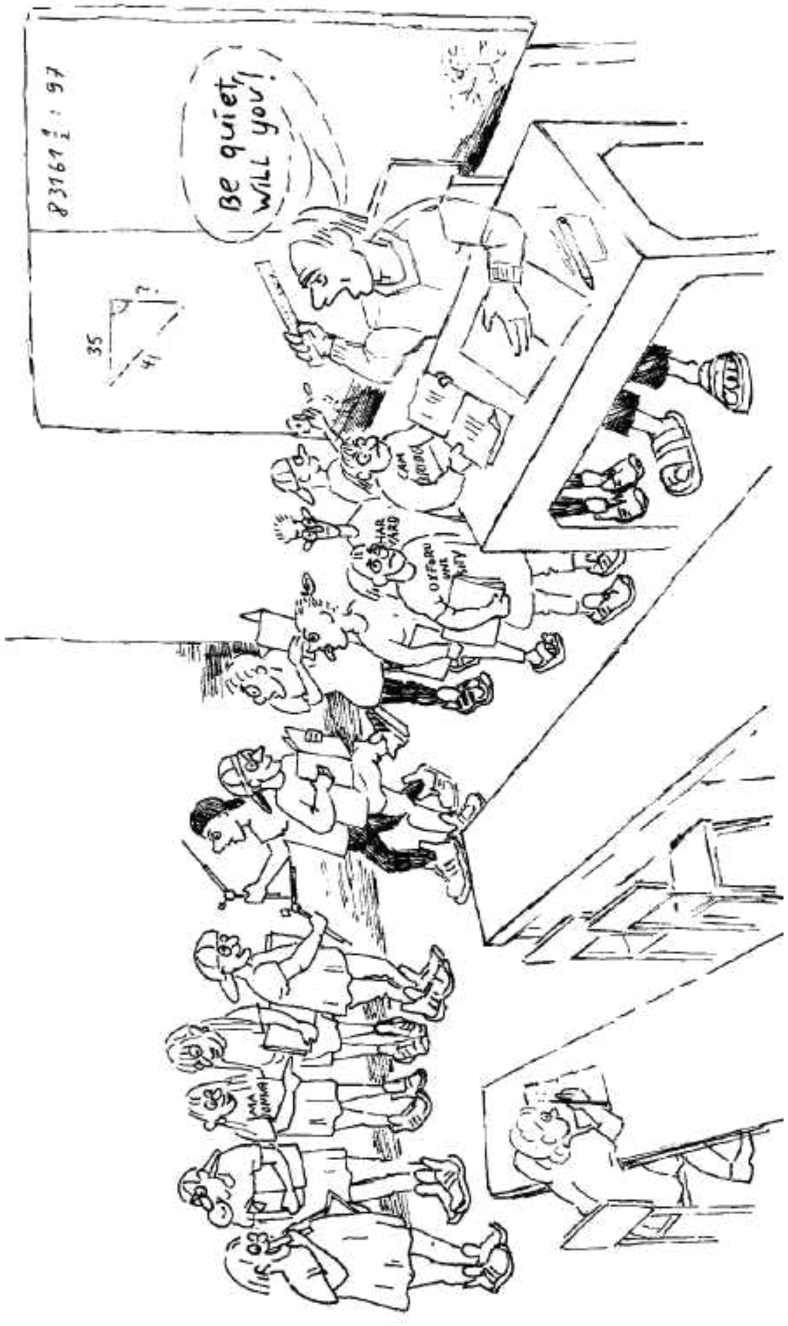

Foreword

by Stephen Sagarin

Can we agree that, from one point of view, math is the most spiritual of disciplines? That, with our efforts, it can bring our thinking closer to conceptual or ‘sense-free’ thinking than other subjects that we study in schools? Mathematics aims, we may say, to bring ideal, eternal truths down to earth. Or, we might also say, it provides us access to Plato’s ‘intelligible’ or ‘knowable’ realm, the world of the true, the good, and the beautiful.

Mathematics, then, deserves tremendous care and attention as we teach it to our students. Rudolf Steiner spent much time in his initial course for teachers (published in three volumes as *Study of Man* or *Foundations of Human Experience*[1], *Practical Advice to Teachers*[2], and *Discussions with Teachers*[3]) discussing developmentally and temperamentally appropriate ways to introduce mathematics in early school years. And, here and elsewhere, Steiner gives clear and pointed advice for teaching, say, the Pythagorean Theorem, later in elementary school. In one instance (*Kingdom of Childhood*[4]), Steiner describes planting a field of potatoes to demonstrate the Pythagorean Theorem.

Two further quotations will suffice to make the point that Steiner’s intention for mathematics teaching was to bring the spiritual truths of mathematics down to this world:

> Your method must never be simply to occupy the children with examples you figure out for them, but you should give them practical examples from real life; you must let everything lead into practical life. In this way you can always demonstrate how what you begin with is fructified by what follows and vice versa. (*Discussions with Teachers*, p.156.)
>
> At first one should endeavor to keep entirely to the concrete in arithmetic, and above all avoid abstractions before the child comes to the turning point of the ninth and tenth years. Up to this time keep to the concrete as far as possible, by connecting everything directly with life. (*Kingdom of Childhood*, p.126.)

Steiner's colleague and mathematician Hermann von Baravalle extended Steiner's work in mathematics teaching over the course of several books and courses. And, since then, sincere, brilliant, insightful mathematicians and mathematics teachers, including but not limited to David Booth, Ernst Schubert, Jamie York, and Ron Jarman, author of the book you currently hold, have extended this work in many ways.

My colleague at Sunbridge Institute, George McWilliam, particularly values Jarman's book. He writes:

> *Teaching Mathematics in Rudolf Steiner Schools* is written out of practice and experience. It particularly challenges and inspires teachers and students in the United States because educational standards and skills in England, where Jarman lived and worked, are generally higher at a given age, perhaps because students are older when they start first grade (class one). Jarman's book provides a comprehensive lower (elementary) school curriculum in math. Paired with A. Renwick Sheen's *Geometry and the Imagination*, a teacher could succeed and even do well with just these two sources, particularly if the teacher is also a student of spiritual science. For teachers who want to dig into it, Jarman addresses the spiritual background of mathematics teaching. Finally, Jarman's book includes good practical advice in the appendices.

An alternative method and culture of mathematics education has also arisen in Waldorf schools, first in the United States, I believe, but now virtually worldwide. This alternative approach began in the 1940s with the introduction in New York City of 'math gnomes.' Since then, it has expanded to include kings and royal families; squirrels and other denizens of the forest; and other anthropomorphised versions of pure mathematics operations and principles. These hover, I fear, unhelpfully between the ideal and the real.

I had been teaching in a Waldorf school for about a dozen years before I encountered math gnomes.[5] They didn't yet exist at the Waldorf School of Garden City, NY, when I was a student and then a young teacher. I then moved to Massachusetts, and was teaching an otherwise very bright girl – she's now a medical doctor – who couldn't divide fractions in seventh grade (class seven). 'I just see gnomes dancing', she said when I asked her what the trouble was. What? It turns out that her previous teacher had explained mathematical operations – including multiplication by the inverse or reciprocal – through the use of blackboard gnomes. The teacher had imparted little or no conceptual understanding, and this poor student was flummoxed.

Since then, somewhat tongue-in-cheek, and yet in utter earnestness and sincerity, I have been on a campaign to 'free the math gnomes'. If you don't believe in gnomes, then why would you introduce them in math class? And if you do believe in gnomes and other so-called 'elemental beings', if you value the real work they do, why would you trivialize them and potentially distract students from a genuine engagement with them by asking them to teach arithmetic to young children?

Because of my experience, I particularly value books like Jarman's, books that are clear, practical, serious, and insightful about mathematics and mathematics teaching. For that reason, I am particularly pleased to write this Foreword and to see this book back in print. It is among the most valuable resources that an elementary school teacher in a Waldorf or Steiner school could have.

I should add that it is not my intention to make anyone feel bad who used or uses gnomes, princes, or animals in mathematics teaching. We are all doing the best we can. I mean that sincerely. A former trustee with whom I worked, to avoid saying that a proposal or practice was bad or wrong, would jokingly say that it was 'suboptimal'. So, when we recognize that our understanding or practice or performance is suboptimal, then we should change it. We don't need to feel bad, we just need to do better. We haven't sinned, we have missed the mark. There's no shame in missing the mark; we all do it much of the time. To hit the mark more frequently and more accurately, however, we study and we practice, we discipline ourselves. Ron Jarman's *Teaching Waldorf Mathematics in Grades 1–8* provides an excellent way for us to introduce math to our students.

Stephen Sagarin Ph.D., 2020
Berkshire Waldorf High School

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2. Steiner, R., *Practical Advice to Teachers* (GA 294), Anthroposophic Press, Great Barrington, Massachusetts, 2000.
3. Steiner, R., *Discussions with Teachers* (GA 295), Anthroposophic Press, Great Barrington, Massachusetts, 1997.
4. Steiner, R., *The Kingdom of Childhood* (GA 311), Anthroposophic Press, Hudson, New York, 1995.
5. An imaginative teaching device introduced by Dorothy Harrer at the Rudolf Steiner School in New York City in the 1940s.

Preface

Alive to what radiates into them through all their senses, young children approach their school life with expectancy. The world is full of interest to them: activities in which they want to participate, all sorts of things to learn about, adults and other children to meet and share with, the vitality and beauty of the world of nature and often its stillness too. Instinctively they feel that gifts from the world's wisdom await them. That their source is something of a mystery and will reveal as yet hidden joys and dangers, enhances their expectant mood. Children have an inner sense that life is a challenge, and a desire to meet it creatively. To help them gain quiet faith in themselves and in what lies deep within them is the task which parents and teachers, acting out of love in the true sense of the word, have before them.

The surrounding world, stimulating in every way though it is, cannot by itself bring knowledge to children. Only their own inner activity, responding to and uniting with what the world presents, can do that. What is it that exists within? An earlier age spoke of the human psyche. Today we can justifiably speak of the soul provided that we can clearly describe its activities – and it has three of them. First we have the impulses to act and to determine the course of our actions, so the soul can affect, indeed change the world about us. Secondly, the world affects us, and to it the soul responds with manifold *feelings* – of wonder, distaste, joy, fear, curiosity, affection, boredom, enthusiasm, envy, gratitude, etc. Thirdly, to whatever we perceive through our physical senses or in inward contemplation of ourselves, the soul responds in *thinking*. This is no automatic reaction. At every moment, in fact, the soul can engender or repress these three activities.

If we pass by a mountain in our travels, for example, our soul can choose to ignore it as a mere lump, or fill itself with wonder at its beautiful shape, its colours, its forested slopes and streams, and feel joy in following the flight of its birds or the clouds drifting past its peak. Whether we also think about the mountain's origin – the weathered result of the slow upward buckling of ancient sedimentary rocks, themselves created through age-long deposits of the bodies of small sea creatures, or ancient plant forms perhaps – is again up to us. The soul *has* to neither feel nor think, if it is so disposed. Nor does it have to initiate action. One traveller may notice smoke and fire beginning to grow at the edge of a wood and do nothing. Another observing it and also thinking of its possible cause by careless campers will take action and inform the local firefighting service.

The human soul, then, is that in us which thinks, feels and wills. It is not of a physical nature. Nor can these three activities be physically perceived. Not even the greatest brain surgeon or cranial investigator assisted by the most refined scientific equipment can see thinking taking place. For thinking activity does not take place in the brain. Certain *results* of thinking, or its absence, are perceptible there, as well as the effects of the activity of our sense organs, but that is another matter. We know that people think, have all manner of feelings and carry out all kinds of will impulses, but we have to go beyond mere physiological investigation to be aware of the existence of the soul.

Educators who have no awareness of the non-physical nature of the soul will be prone to tackle their work with the positivistic, reductionistic attitude developed in the modern theoretical, materialistic sciences. They will also not be able to understand the nature of the individuals before them. We are all aware that we possess a unique individuality, whose existence we acknowledge whenever we use the word 'I'. But how this is related to our lives within our earthly bodies has to remain an unfathomable mystery if we cannot understand that the soul, though not material, is nevertheless absolutely real. The individuality, the 'I', the human spirit (all equivalent terms) exists within us as the captain of this soul.

In many of life's decisive moments this spirit of ours has to determine how much to rely on what its three lieutenants variously advise it. To rely solely on one of them may be to court disaster. Take the case of marriage. To be swayed only by emotional attraction and compatibility of sexual urges – part feeling and part willing – will lead to the wish for a new, different partner before many months have passed. On the other hand, sole reliance on a clear critical assessment of the proposed partner's talents and activities (similar interests, health, educational background, artistic or sporting talent, wealth, temperament, political views, ability to cook or mend furniture and so on) could be equally disastrous. The heart's feelings – the real deep feelings of the soul – need to be listened to first, but clear thinking about what being married to such a partner is going to mean in the future is also essential. Yet something more is also required – a spark of will which one can trust. Only the captain can make the final decision. Only decisions made by the 'I' in clear consciousness, helped by the three soul activities, will give grounds for hope of a successful outcome.

In Britain's most critical moment during the Second World War Winston Churchill quoted from W.E. Henley's poem[1] in which the approach of death is contemplated. The poet had surveyed his successes and failures, his good and bad deeds, his thoughts, feelings and actions as a whole. The verse Churchill quoted well illustrates the role of the human individuality.

> *It matters not how strait the gate,*
> *How charged with punishments the scroll;*
> *I am the master of my fate,*
> *I am the captain of my soul.*

Among the physically invisible aspects or entities of the human being referred to above, there is another which can be named our *genius*. Unlike descriptions of things or beings in the physical world, where no one body can occupy the same space as another, these inner entities are fluid and can flow into and out of each other. What is called genius (which is certainly not the product of physical genes) has qualities of a soul nature but can be perceived as mentor and guide to the ego or soul captain. This genius has an intelligence much deeper and more widely embracing than what we call our intellect, but can also inform it, just as it can nurture other parts of our thinking activity and can penetrate our artistic feelings and willed actions. It may reveal itself as literary, scientific or artistic genius. It is there in everyone, but is frequently dormant; and when it does stimulate us and help develop our creativity we tend not to use the term 'genius' unless the creativity is of very high order.

With all the foregoing in mind let us turn to the subject of mathematics. *In every person, every child, there lives a mathematical genius.* Never should a teacher even think (let alone say), 'This particular child will never understand maths. It's beyond the talents which the gods (or his genes – if you want to be materialistic) have endowed him with.'

The genius is asleep to begin with, and rests within the beat of heart and lung – the true womb of arithmetic – and in the bones and muscles of the limbs – the true womb of geometry. Through encouragement, especially in the singing of songs and the making of music, one part of the genius can rise up to the level of the larynx and collar bone and begin to dream. The other part can move along the arms to the hands when a child is helped to draw in a colourful and healthy way; it too begins to dream, this time a geometrical kind of dreaming. When the two parts or aspects ascend to the head, later to unite, the full genius can awaken and mathematics can become a conscious activity possessed by human thinking.

What role mathematical activity plays and can play vis-à-vis the sciences of the physical world on the one hand and the sciences of the soul and spirit on the other will be dealt with in the following chapters.

It may seem an utter contradiction to maintain in one paragraph that the soul's activities are non-physical, non-material and then in a later paragraph to refer to parts of the human body which allow the equally non-physical genius to rise from sleep, passing through other bodily regions as it first dreams and then awakens. But just consider the word 'heart'. We use the word, on the one hand, to denote the physical organ that controls the pulsing blood in us. Has it not happened to many of us, when meeting someone we are physically attracted to, that our blood speeds up in response to our perceptions? We can be grateful to the heart that it prevents the blood beat speeding out of control. But hold on a moment. Is this really just a physical matter? We can learn to control, even master our instincts. In every situation, any activity in which we may find ourselves becoming engaged, not just love affairs, we can ask ourselves, 'Is our heart really in it?' This does not refer to the physical heart. Nor is it merely a useful metaphor. The genius of language understands that as well as a physical heart we have an invisible, non-physical heart, which has an intimate relationship with the anatomical one. So too, when we use words like 'hands' and 'head', it is not necessarily just the physical parts of our body we are indicating. It is via the invisible head, heart and hands that the human soul and human spirit can mediate with and control our physical body.

May it not be that at some future time in evolution – provided we have worked for it in the right way, with the help of the powers with whom our genius may put us in touch – we shall even be able to begin to control our liver and all other organs, too? At such a time the work of doctors and hospitals would become very different. To say, then, that in becoming mathematically active our genius has risen from heart to throat to head is neither a physical nor just a metaphorical reference. How this movement can be taken account of in the practical teaching of mathematics will be shown in the chapters that follow.

In pursuing such considerations of the invisible (spiritual) but essential activities taking place in the human being, a whole science of the spirit can be evolved. This was what, in fact, Rudolf Steiner[2] did in the first quarter of the 20th century. All that is developed in this book owes an immeasurable amount to the inspirations for further research which he made available at that time.[3] This science of the spirit is also known as anthroposophy, the essential basis for which is to be found in Steiner's fundamental book *The Philosophy of Freedom*.[4]

Before attempting to teach mathematics, whether this be arithmetic, algebra, geometry, trigonometry, calculus, computer programming or chaos theory, it is important to ask what sort of subject it really is. To this the first and second chapters will be devoted.

The aim of all teachers of mathematics is to bring about a confident capacity in their pupils to be able to move freely within the particular realm that is being focused upon, be it the life of number, the emergence of geometrical form and its metamorphoses, or the applications of mathematical ways of thinking to practical situations and technical tasks in the world. Over 40 years' teaching experience of the subject have amply confirmed for me that when children and young people have the good fortune to be taught in a single school from the age of six or seven to 18 by a small number of teachers whom they learn to know deeply (and vice versa), progress can become wholesome and stable. Such conditions obtain primarily in the Rudolf Steiner (Waldorf) schools, which were founded in 1919, and of which over 700 now exist spread through all continents of the world. But the realisation of the importance of not changing children's teachers at the end of every year or so is found in many other good schools, too. In Waldorf schools this realisation is complemented by teaching maths in block periods of roughly a month every term for the first two hours of the school day, in addition to having regular practice lessons of short duration in the weeks when the block period – called 'main lesson' – is operating in other subjects such as English, science, geography or history.

This book aims to present helpful, practical ideas and suggestions for mathematics teaching in school – or indeed for anyone who may wish to relearn what he learnt, or failed to learn, in his own schooling. The focus, however, is on the way teaching can be developed in a Waldorf school. It is to teachers pioneering in these schools, who in many countries labour with inadequate salaries (Britain at present offers less funding support for such schools than any other government in the world – zero at the time of writing), that this book is dedicated.

Important note to class teachers – especially in Steiner Waldorf schools

The treatment of all the mathematical topics applicable to the 7–14 age range in the chapters ahead is a comprehensive one. Very few of the topics that can justifiably be introduced to children over these eight years have been left out. This does not imply that *all* the topics that *are* included must be dealt with by a teacher to achieve an adequate mathematical education. Many of them certainly are essential, but it has to be left to the individual teacher to select which developments will achieve optimum progress for the particular group of children he or she is responsible for. Every school will have its own recommended mathematical curriculum for each age of childhood. To what extent this parallels the curriculum outlined here (see Chapter 10) or the national curriculum propounded by a government department is a matter for that school and its teachers.

Both the suggestions for curriculum and the examples for children to work at are made with a very wide ability range in mind. The way in which they are introduced here will show, however, that every child from the apparently most innumerate to likely future university graduates in mathematics and the sciences can benefit from a broad variety of numerical and geometrical experience. The criticism that much of the work in the chapters ahead is too difficult for an 'average child' is invalid. It misses the point. Experience shows, however, that such criticism will continue – often arising through fear on the teacher's part that his own ability is insufficient to grapple with all but the basic mathematical topics in primary schools.

Before we get to grips with the nitty-gritty of maths teaching, however, let us first look at how mathematics has been perceived through the ages, particularly in regard to its spiritual significance. As I hope will become clear in the course of this book, this aspect is one that, albeit indirectly, can and should inform even the most basic levels of mathematics in schools.

Introduction: Mathematics and the Mystery Schools

§1. Greek teachings

The previous practice and knowledge of mathematical activities was a prime requirement for students (called novices in those days) seeking to enter Greek mystery schools. Plato declared, 'God geometricises'. His predecessor Pythagoras required his students to work in mathematics for most of the time in their first-year course, before subsequently being initiated into the world's deeper mysteries – Where have we come from before being born? What happens to us after death? What is the origin of our earthly home? What do the stars reveal? How can one be trained to hear the harmony of the spheres? When are the best times to open the inner ears to listen to the conversations of the gods? Why are we at first unaware of our true tasks and destiny in life?

There are no extant records of the Pythagorean school written by its participants, but several convincing stories have come down to us through various biographers, foremost among whom is Iamblichus.[1] Eduard Schuré has presented an imaginative account of what took place, based upon spiritual perception.[2] More recently Ernst Bindel, a mathematics teacher in the first Waldorf School in Stuttgart, southern Germany, has written an enlightening book on the mathematical side of Pythagoras' work and teaching.[3] From studying these and other books and developing inner sources of perception – again through the good offices of one's genius – the following description emerges.

A novice wishing to enter the school in Croton (whose ruins are to be found on the Calabrian coast of modern Italy) had first to give away all his possessions to others or, if he so wished, to the school. He was then required to attend an interview with the one we would nowadays call the principal of this college or university. Pythagoras' first question to him would be, 'Can you in a particular class, whose membership is solely determined by the chronological (as distinct from developmental) age of the children, there is plenty for the cleverest and weakest to get their teeth into. Never need it happen that a weak child or their parents should feel that more is being demanded than the child can cope with. Equally unnecessary is it for a talented individual (or again the parents) to complain that they haven't enough work to do of a challenging kind. Some of the best schools in the earlier part of the 20th century in Britain were the village schools. Only a few now exist. The teachers had a big age as well as ability range to deal with. In many cases they developed fine, imaginative presentations. Younger children could listen in to lessons given to older ones and vice versa, and the content was the same for both the more gifted and less able children. We just don't need the strait-jacket techniques of streaming any more than we need rigid national curricula. But fortunately, with good sense to sift out from the latter what we can use, and with imagination to transform it, we can invest the substance of our teaching with wholesomeness even within such constraints.

There is no limit to ways of stimulating children in mathematical activity. It is only when we lose sight of the essentially human, fail to read child nature and its inner development at each age, that we don't choose the best subject matter or fail to find healthy imagination in ourselves to encourage their learning. Dull theoretical or unsuitable technological influences then creep in. There is a simple way of finding out whether we are succeeding or not. The children will tell us by their eyes, the way they breathe and smile. When at the end of the main lesson block, whether it is maths or some other subject, they extravagantly and maybe shortsightedly declare, 'This is the best main lesson we've ever had', you know you have not failed to stimulate them.

Chapter 3:

Suitable Examples for Children's Written Arithmetic in Classes 1 to 3 (6 to 9 Years Old)

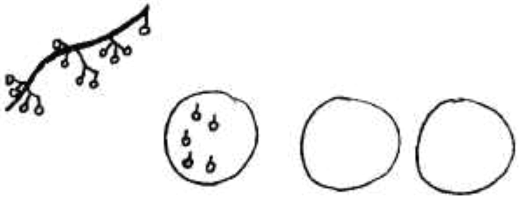
§1. Class 1

Several examples have already been given in Chapter 2. Since children are not yet able to read words at the beginning of this class, questions need to be pictorial in form. Careful copying from a blackboard is in itself a real exercise. Before they do this it is good to look through with them all the questions on the board which you have set for the class that morning. Make sure they know what you want them to do in the whole set before they copy the first question; also make sure the board drawings are large ones.

Demonstration type

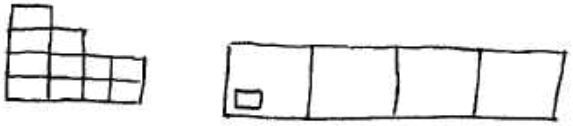
Each child can have a little basket or box of small nuts or stones from which to extract and arrange piles demonstrating the sums. Then by copying these arrangements they can complete their drawings and write the appropriate Roman numerals below them. The nuts and stones can naturally represent anything – oranges, jewels, people, animals, money... The use of freely chosen colours will enhance the drawings. The teacher's descriptions and explanatory words are indicated in brackets.

1. (apples, picked and placed on these plates – choose how many you like for the other two plates, but don't leave any unpicked. Put numbers instead of dots underneath.)



• = • + • + •

2. (bales of hay for cattle in four fields)



• = • + • + • + •

3. (eggs put into three baskets)

• = • + • + •

4. (each box has to have the same number of caterpillars)

• = • x •

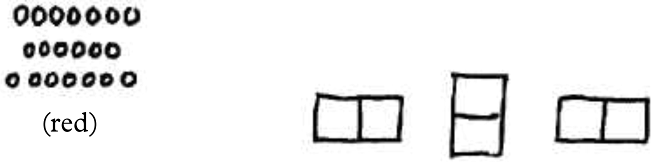
5. (each part of the sky has to have the same number of stars)

• = • x •

6. (put these rings into as many triangular boxes as you like, but each box must have the same number of rings)

• = • x •

7. (use the red spots to make dominoes where each domino is a double)



• = 2 x • + 2 x • + 2 x •

8. (arrange all these flowers in vases so that each vase has one more flower than the one before)

• = • + • + •

All these examples, it must be stressed, are not intended to be photocopied for children, nor even just copied from this book on to the blackboard. They are here to stimulate the teacher's own imagination. They will find their own contexts and produce much more interesting examples.

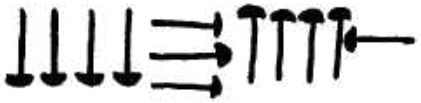
Imagination type

Here we dispense with nuts and stones. The children have to use their own fingers and their imagination. The only drawing to be made each time is the one copied (and perhaps made with greater beauty). This represents the *given* element (what is seen). The goal or result is shown in Roman numerals. The child has to count what is seen and then decide what has to be done, expressing each result also in Roman numerals. The two examples in Chapter 2 could be complemented by two more in the form of a completed birthday story. Here is another story.

1.(A carpenter took these planks out of his store to make a shed, but found that he needed 19 altogether. How many more did he fetch from the store?)

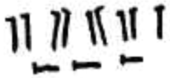
XVIIII = +

2. (He brought these bolts with him but only used 8 of them. How many did he return to his store?)



VIII = –

3. (He had these tacks to fix pieces of felt to the roof, but each piece needed 6 tacks. How many pieces of felt could he fix?)



VI = ÷

4. (The floor needed 20 beams. He could carry this many at once. How many journeys did he have to make from the store?)

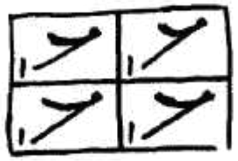
XX = x

Smaller numbers could be used in such examples to begin with, increasing their size in later examples for the cleverer children.

Computation type

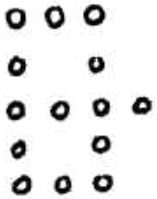
A few questions using Arabic numerals in the imagination type of sum might be done at first. Then pictures as well as Roman numerals can be dispensed with altogether.

1. (How many pages of stamps do you need to stick one stamp on each of 24 letters to be posted?)



24 = x

2. (I only need 8 of these pound coins. How many can I give to my friend?)



8 = –

3. 11 = 3 +
4. 5 = 10 ÷

Then come the sets of questions on the four rules in a purely computing manner. Remind children that question 17 can also be read as 'How many threes in six?'

5. 2 + 7 =
6. 4 + 3 =
7. 7 + 6 =
8. 14 + 4 =
9. 6 – 3 =
10. 9 – 4 =
11. 14 – 6 =
12. 20 – 15 =
13. 4 x 2 =
14. 3 x 3 =
15. 3 x 5 =
16. 6 x 3 =
17. 6 ÷ 3 =
18. 12 ÷ 2 =
19. 17 ÷ 1 =
20. 16 ÷ 4 =

Later on a teacher could write a sum on the board (still in Class 1) but cover up one number with their hand and ask the children to say what it must be before removing their hand. Then with another sum the teacher could use the backs of their wrists to cover up a number. With a third sum they could just draw their wrists, and with a fourth write x. Children could then do sums where they have to replace x by a number. When teaching algebra in Class 7 it is good to be able to say, 'Actually you learned to do algebra in Class 1'.

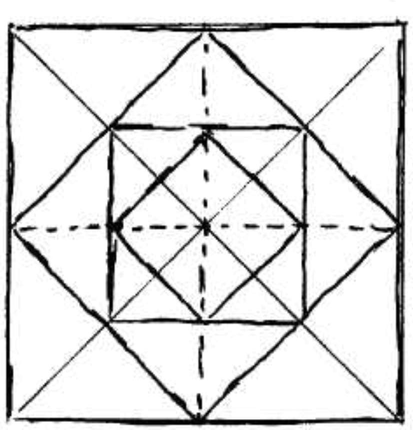
So: 6 + 2 = x   x =   (What is x?)
3 x x = 12   x =
20 ÷ x = 5   x =
x – 9 = 4   x =

There are, of course, many ways of introducing written sums in Class 1. A teacher might say to the class, '8 swallows were sitting on a telephone wire, then 3 flew away'. The teacher writes 8 – 3 = on the board and asks everyone to copy and complete the arithmetical sentence to show how many swallows were left on the wire.

To bring in additional variety it is good to consult various textbooks in public libraries, to look at examples suggested in National Curriculum publications and – most helpful of all – to look through the written work done by children in previous Class 1s, including those in other schools. There follow some examples of wider range – needing oral input from the teacher.

1. Copy and write down the neighbours (above and below) of 35, 19, 50.
2. Can you do those in Roman numerals, too?
3. Make a list of the numbers between 10 and 20 which don't come in the 2 times table.

4. 'Guess how many marbles there are in this glass. Write down your guess.' 'Now, count them', you say to one child, who empties the glass, marble by marble. 'Now write down by how many you over- or underestimated the total.'
5. Draw a triangle and mark the middle of each side. Join up these middle points. How many triangles are there in the whole drawing?
6. Draw a square and its diagonals. How many triangles *altogether* can you count now?
7. Write down all the Roman numerals you can make using just 7 sticks. How many did you find? Which is the highest number?
8. How many more numbers can you find if you can use at most 6 and at least 1 of the sticks?
9. Draw a clock with Roman numerals on it. Now write outside each number another one to make it a 24-hour clock.
10. Draw a clock with modern numbers on it. Join up all the numbers that come as answers in the 2 times table with red lines and all those in the 3 times table with blue lines. Have any lines become purple?
11. Suppose you were to share out 28 crystals among a group of girls so that the oldest received 1 crystal, the next oldest 2 crystals, the next oldest 3 crystals and so on. How many girls would have to be in the group?
12. See if you can draw this picture carefully, using different colours for squares and diamonds. Then draw one more square inside and one more diamond outside.



To do these questions, children will need to use their fingers, or in some cases small baskets of nuts or stones.

Some teachers whose Class 1 contains a preponderance of slow learners, and also some teachers of an older school of thought in Waldorf schools, may consider that some of the questions in this section are too advanced or too abstract or intellectual for Class 1. Whilst one can sympathise with such a view – for certainly the danger is always present of going beyond what the nature of a seven-year-old requires from us – the following remarks are in place here:

i) 'Intellectual' in Rudolf Steiner's use of the word simply means analysis or going from the whole to the parts;

ii) It is only when one stops with these parts, ignoring their origin (a failure in modern particle physics), and fails to use the synthesising power of reason to re-attain the whole in a more comprehensive way, that one may be justified in using the word 'intellectual' in a derogatory way.

(iii) 'Abstract' means to take something out of its context, for example quoting from a lecture, usually without fully understanding its significance in the whole lecture.

§2. Class 2

Besides the frequent, diverse oral practice of all the multiplication tables and number bonds in addition and subtraction, examples of the following kind need to be done regularly in written work. The list below might be regarded as an hour's revision test towards the year's end.

1. $\begin{array}{r} 35 \\ \underline{+24} \end{array}$
2. $\begin{array}{r} 718 \\ \underline{-213} \end{array}$
3. $\begin{array}{r} 32 \\ \underline{\times 3} \end{array}$
4. $2\overline{)128}$
5. 424 + 3 + 31
6. 58 + 26 + 13
7. 375 – 158
8. 153 x 4
9. $5\overline{)5125}$
10. 616 + 4127 + 358 + 91
11. 15006 – 1297
12. 142857 x 7
13. $12\overline{)379248}$
14. 2166 x 27 (multiply 2166 by 9 and then the answer by 3)
15. 33792 ÷ 44 (similar method)

16. Subtract 29 from 917. Then divide the answer by 7, showing the remainder.

17. Work out the 13 times table.

List of answers:

6
13
26
39
52
59
64
65
78
91
96
97
104
117
126
130
143
156
217
458
505
612
768
888
1025
5192
13709
31604
58482
999999

Since so much depends upon the teacher composing and working out simple examples of the four rules each evening before the lesson, further examples of this kind for Class 2 need not be given here. But notice the value of writing on the board along with the questions, but not in the same order, a set of answers. Children will notice if their answer to a question comes somewhere in the list. If so they don't need to ask you if their sum is correct and much time and energy is saved for helping children who are having problems.

A beginning can be made in Class 2 with written problems, first copied from the board, then read aloud by the whole class, then by one or two individuals weak at reading, then discussed between teacher and class. Indications written on the board by teacher or child might follow, then everyone can get down to the task individually in their own book.

Two examples:

1. If a hundred and sixty-five tadpoles were shared equally among five tanks, how many tadpoles would live in each tank?

2. The swan carried the princess for thrice four weeks across the great sea. Seventy-two times did she see the sun rise in her journey, but the other mornings were dark with clouds. How many dark mornings were there?

Practice in changing numerals and large numbers into words and vice versa belongs to Class 2.

§3. Class 3

It is important that before children reach the age of 10 they learn to handle whole numbers with at least 3 or 4 digits in each of the processes of the four rules (or four operations). This means that they should have achieved some proficiency in long multiplication and long division. The class teacher should try to ensure that at least two thirds of the children have managed this by the end of Class 3. For looking ahead, the main work in Class 4 mathematics will concern fractions and decimals. Weakness in handling whole numbers could well be a serious hindrance in learning about these new number experiences. The examples below (answers in brackets) indicate a progression from easier to harder sums.

1.	312	x	21	(6552)	– no ‘carrying’.
2.	1102	x	43	(47386)	– no ‘carrying’, but use of zero.
3.	24	x	42	(9828)	– a little ‘carrying’.
4.	574	x	67	(38458)	
5.	97	x	85	(8245)	
6.	234	x	567	(132678)	
7.	9999	x	111	(1109889)	
8.	1471	x	143	(210353)	
9.	Work out the 53 times table.*				
10.	1272	÷	53	(24)	
11.	2769	÷	71	(39)	
12.	2002	÷	22	(91)	
13.	69741	÷	123	(567)	
14.	50001	÷	23	(2173, remainder 22)	
15.	100000	÷	101	(990, remainder 10)	

* (53
106
159
212
265
318
371
424
477
530)

When children spot shortcuts, e.g. in question 12 divide by 2, then 11, or in question 15 don’t write out the 101 times table – certainly allow this. There is little value in dividing or multiplying by a four-digit number, but you might have a couple up your sleeve for clever ones who soon rattle through the questions set for the class. It is also instructive to get children to do the reverse calculation, so from question 5 do 8245 ÷ 85 and from question 13 do 567 x 123. Learning about measurement gives rise to a big range of problems and this provides the opportunity to give a lot of practice in problems written in sentences, as distinct from pure number problems. On the simplest level you can ask the children to fill in the spaces:

1. 5 weeks = days
2. 3 years = calendar months
3. 63 days = weeks
4. 120 seconds = minutes
5. 3 days = hours, and so on

You can also do similar things with liquid measure, length and weight. When it comes to questions involving whole sentences or paragraphs, then it is best to photocopy a set of such question in your best handwriting and let the children later paste these into their own main lesson books alongside their arithmetic. Here is a range of such questions.

1. Mary went for a 4-day hike. She walked 10 miles the first day and it took her 5 hours. Next day she walked 11 miles and 2 furlongs, taking her 5 hours and 20 minutes. On each of the last two days she walked 12 miles and 3 furlongs, taking 6 hours and 20 minutes each time. How *far* did she walk altogether?

2. How much *time* did Mary spend walking altogether?

3. Each of eight big families ordered 1 gallon and 3 pints of milk from the milkman. How many gallons altogether did he have to supply?

4. I went into a confectioner’s shop and bought:
    3 loaves costing 83p each,
    4 packets of biscuits costing £1.20 each and
    2 cakes costing £1.95 each.
   What change should I get from a twenty-pound note?

5. The floor of a building is 15 yards long, 12 yards wide, and it has to be covered with square concrete slabs, each side of every slab being a yard in length. How many slabs must be used?

6. If one of the slabs were marked out in inch squares, how many of these inch squares would there be altogether on this slab?

7. Three heavy men weighing 15 stone 12 pounds, 16 stone 9 pounds and 17 stone 6 pounds stepped carefully into a light boat which would sink if more than 50 stone were placed in it. Did the boat sink?

8. Robert bought 25 coloured pencils each costing 35p. His change from a £10 pound note consisted of a small coin and six slightly bigger ones of a different shape. What were all these coins?

9. How many centimetres are there in 5 kilometres?

10. A roll of cloth 70 yards long has to be cut into pieces 8 feet long. How many pieces will there be and what length of cloth will not be used?

11. If the roll were 70 metres long instead, and pieces 2 metres 60 centimetres long were required, what would the two answers be then?

12. A cubical tin in which every edge is 10 cm long will hold a litre of water weighing a kilogram. If a quarter of the litre is poured away, what will the remaining water weigh in grams? What depth of water in millimetres will there be now?

Naturally, simpler examples of the above need to be set first, for the reasons given in Chapter 2. So if the morning's written work is about length – and we will suppose that metric lengths will be introduced later in the month's main lesson block – you might have:

1. How many inches are there in 2 feet?

Later,

7. The fields of a farm all ended along the bank of a straight canal and each field was a furlong in length. Altogether the canal ran along 2½ miles of the farm. How many fields were there?

Later,

13. A goods train had a diesel engine 4 yards long and 33 trucks each 19 feet long. Every coupling space was a yard. The whole train was longer than an eighth part of a mile. How much longer? Give the answer in yards.

As usual, include the three answers in a list below the questions, i.e. 20, 24, 26. It doesn't matter at all that the weaker children will not have time to get anywhere near question 13. They will feel, however, that if they had a much longer time, they would probably have been able to do that question too, with a little help. Next morning, though, they will watch the teacher doing question 13 on the board, helped by suggestions from the class, as revision of the work of the day before. Some of the cleverer ones may be inspired by the beauty and clarity of the teacher's layout on the board in comparison, maybe, with their own. Once again the social divisiveness caused by making different ability groups work at different assignments, in turn brought about by examination league tables and other apings of professional football, has to be avoided.

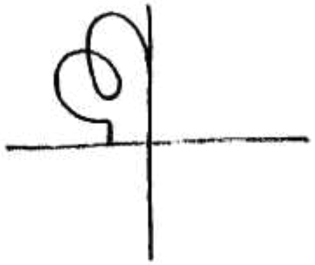
§4. Practice periods

It is important, certainly from Class 3 onwards, that each week, in addition to main lessons of up to two hours daily, there shall be two or three 40-minute 'extra main' or practice periods taken by the class teacher. During a maths block of main lessons they will be devoted to practice of the English language. During other blocks these shorter periods need to be used for arithmetic (and drawing exercises). Here it is not a case of introducing new topics but of practising what has been taught in earlier maths blocks. For learning maths is like learning foreign languages, which also need regular practice.

So later in Class 3, such a 40-minute slot would include a five-minute introduction by the teacher, 30 minutes quiet individual work by the children and a five-minute ending for them to tick or cross their answers whilst the teacher calls out the correct ones. The assignment may be:

1. Add up 329, 41 and 106.
2. How many gallons are there in 56 pints?
3. A collection box for a charity contained the following coins: 21 pounds, 35 x 50p, 52 x 20p and 17 x 5p. What was the total money collected?
4. How much less than a million is 7 times 142857?
5. Subtract 16m 74cm from 19m 39cm.
6. In how many of the 12 multiplication tables does the answer 36 come?

7. Copy and add more lines so that every two spaces are balanced by the line between them.



8. Fill in the blanks in this addition sum:

$$\begin{array}{r} 257 \\ +\ \cdot\,18 \\ \hline 38\,\cdot\,\cdot \end{array}$$

9. The first day of a certain leap year was a Sunday. Every day in that year except Sundays I put 5p in my savings box. How much had I saved by New Year's Eve in that year?

10. Divide 67,081 by 259. What do you notice?

11. Each winter day a farmer cleaned out 2 hundredweight of dung from his cow stalls. How many tons of dung did he clean out in November?

12. **RON** with each letter 'turned round' looks like this: Do the same separately for each digit of 8619. Your four-figure number will now be bigger. How much bigger?

Answers (except for question 7 and 8):
1, 2-65, 3, 5, 7, 15-65, 49-75, 259, 297, 476.

The children could do these questions in any order they liked. Afterwards you could invite them to do others for homework, but let this be quite optional at this age.

Chapter 4:
The Heart of Childhood

§ 1. Child development

When we review the school life of children up to the age of nine, the following picture emerges:

Spiritual scientific phases	Soul emphasis	Class & age	Child 'says'	Waldorf curriculum indications
Physical body complete in basic structure as its building process reaches beyond the age of six.	–	Kinder-garten	Love me along with other children.	Creative play and imitation.
Physical body adjusts to physical surroundings. Building force is released to develop habits, memory, rhythms (ether body).	**W I L L**	I $_7$	Bring rhythm and good habits into my play and my store of fantasy.	Writing. Fairy stories. Numbers and the four rules.
Settled posture (ether controls physical). Astral presentiment in etheric body.		II $_8$	I feel at ease in my actions. Teach me about courage, cleverness and worthy goals.	Legend & fable. Multiplication tables.
Ego presentiment in etheric body. It practises control of life forces.		III $_9$	Who are you as a being of will? Are you really acting for my benefit? Whence comes your authority? What do you know about practical life?	Old Testament stories. Farming, dung, building, measurement.